Behavior of Suspended Matter in Rapidly Accelerating Viscoelastic Fluids: The Uebler Effect

A. B. METZNER

University of Delaware, Newark, Delaware

The observation that large bubbles entrained in accelerating viscoelastic fluids may suddenly stop and remain stationary for long periods of time, although imbedded in a region of high fluid velocities, is noted and described. An analysis of this phenomenon, termed the *Uebler effect*, shows that it may be expected to occur with all particulate matter provided that the continuum field fluid is accelerating sufficiently rapidly to generate high stresses as a result of fluid stretching, and provided these stresses change sufficiently rapidly in the direction of the velocity vector in the flow field.

The behavior of suspended gas bubbles or particles is one of reasonably broad interest in engineering in general and in chemical engineering transport processes in particular. Furthermore, the use of small bubbles as tracer particles with which to follow fluid motion, long a standard but intermittently used research tool, has received significant impetus through development of the *hydrogen bubble method* (*3, 6*) for the reproducible introduction of very small bubbles; its use has recently been extended to viscoelastic fluids (*7*). Thus the behavior of bubbles and particles is of both research and engineering interest, and the conditions under which they follow (or do not follow) the velocity field are of significance.

It has been shown in a recent thesis (*11*) that small bubbles (diam. $\approx$ 0.05 cm.) may be used as tracer particles to study moderately accelerating flows of both Newtonian and viscoelastic fluids; the flow rate, as determined by using the bubbles, checked the volumetrically measured flow rates within a few percent, and all deviations were random. However, when much larger bubbles (diam. $\approx$ 0.5 cm.) were introduced into the accelerating velocity field (flow from an "infinite" reservoir into a much smaller tube having an I.D. of 1.48 in. or about six to eight times the bubble diameter), a new phenomenon was observed: the large bubbles move through the upstream reservoir with no apparent unusual behavior but, to quote Uebler, "stopped abruptly just prior to entering the pipe, as though some mysterious force reached out to hold them," while the immediately adjacent small bubbles proceed on their courses. The large bubbles may remain stationary for periods of a minute or more and then suddenly release and proceed on their way. A typical photograph of such a stationary bubble is shown in Figure 1.

Having thusly defined the *Uebler effect* as the sudden stopping of large bubbles in an accelerating velocity field, the purpose of the remainder of the paper is to present an analysis of this phenomenon.

ANALYSIS

Consider the large bubble as depicted in Figure 2 in a generally accelerating flow field having streamlines of the form indicated by the numbered lines. The terms $(\tau_{11})_A$ and $(\tau_{11})_B$ denote the total normal stress terms at sections A and B, respectively. Because the streamlines are curved, these finite normal stresses exert a hoop tension F on fluid elements nearer the center line; this force has a finite component in the upstream direction, the magnitude of which may be calculated by using the r component of the stress equations of motion (with inertial and gravitational forces neglected) as

$$\frac{\partial F}{\partial r} = \frac{\partial}{\partial r}(-\tau_{rr}) = \frac{2\tau_{rr} - \tau_{\theta\theta} - \tau_{\phi\phi}}{r} + \frac{1}{r \sin\theta}\frac{\partial}{\partial\theta}(\tau_{r\theta}\sin\theta) \quad (1)$$

Herein τ_{rr}, the r component of the total stress, may be equated to the negative of the force F, since the usual rheological convention is to take stresses as positive when they are tensions rather than pressures. Because of symmetry the ϕ derivatives have been set equal to zero; no other direct or implicit assumptions are required to write Equation (1) in the form given.

In the case of large bubbles the interface is essentially free (*2*), and the $\tau_{r\theta}$ term is zero in the vicinity of the bubble. Thus, its derivative *w.r.t.* θ must be zero, and this term of Equation (1) may be dropped. In the case of small bubbles this term may be finite, but it will be of comparable magnitudes in viscous and viscoelastic liquids; that is, it does not operate in any manner which would serve to stop the motion of the bubble. The normal stress terms τ_{11} may, however, be many orders of magnitude greater in accelerating viscoelastic fluids than in Newtonian liquids; an element of the fluid is being

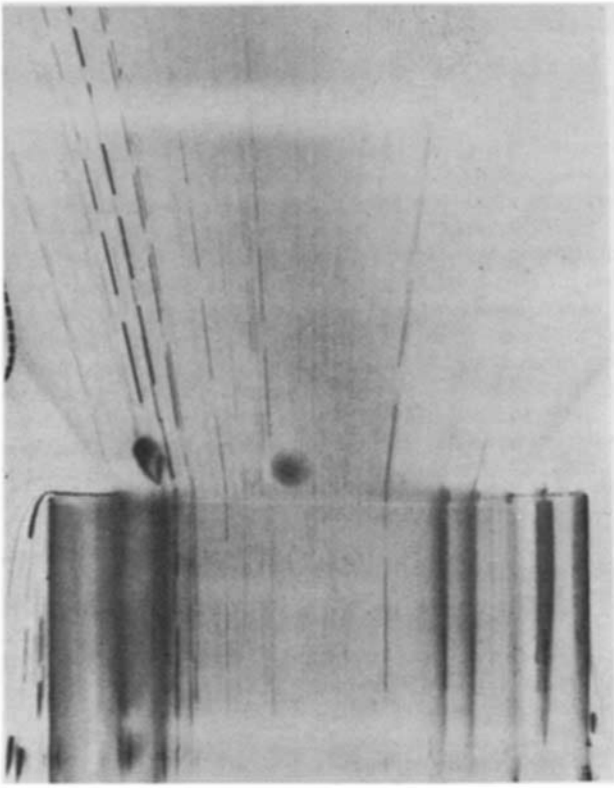

Fig. 1. Photograph of bubbles illustrating the Uebler effect. Illumination confined to a thin vertical slit ⅛ in. wide through central plane of tube, so that all bubbles visible are in this plane. Fluid: 0.45% solution of Separan, AP-30, a partially hydrolyzed polyacrylamide of high molecular weight. Flow conditions: mean velocity in tube approx. 4.0 ft./sec. Exposure time approximately 0.01 sec. Streaks formed by small tracer bubbles show extent of fluid motion during this time period. Edges of large bubbles fuzzy as they oscillate and vibrate, but note absence of motion of the magnitude experienced by the small bubbles.

stretched as the flow converges, and the contributions arising from this kind of deformation may be very large.

Consider the stretching of the element of fluid contained between streamlines 1 and 2, at position B as shown in Figure 2. To calculate the stresses involved a constitutive equation must be chosen to describe the fluid properties; the convected Maxwell model used previously and known to be both a simple and a reasonable approximation (*5, 12*) will be employed:

$$\underline{\tau}' + \theta \frac{\mathfrak{d}\underline{\tau}'}{\mathfrak{d}t} = 2\mu\,\mathbf{d} \tag{2}$$

Here, θ and μ denote the relaxation time and viscosity of the fluid, $\mathbf{d}$ the deformation rate tensor, and $\underline{\tau}'$ the stress tensor relative to an arbitrary hydrostatic pressure term. The symbol $\mathfrak{d}/\mathfrak{d}t$ denotes the Oldroyd contravariant convected derivative (*4, 12*).

To obtain a first approximation to the stress terms in Equation (1) by means of Equation (2) the following assumptions are employed:

1. The geometry of the stretching element of fluid in the converging velocity field may be taken as equivalent to that of a flat sheet. That is, the curvatures in the θ and ϕ directions are neglected during calculation of the stresses on the right side of Equation (1), although not of course in calculating $\partial F/\partial r$.

2. Partial time derivatives are zero. This assumption restricts the analysis to the steady flow conditions encountered when the bubble is stationary as shown in Figures 1 and 2. Any analysis of the deceleration of the bubble as it first approaches the tube entry is omitted.

3. The stretching process, in the region of interest, occurs at a steady stretch rate $\partial v_1/\partial x_1$. This assumption is motivated by the experimental observation that the converging velocity field, in the absence of any bubbles, tends toward a constant stretch rate of a magnitude determined by the upper limiting stretch rate to which a given viscoelastic material may be subjected.

The above assumptions, although not correct in detail, appear to be reasonable first approximations which may be improved upon subsequently. With these used, the problem is reduced to one of the stretching of a series of flat sheets of fluid and of the summing of the resultant forces by means of Equation (1). The tensile stresses set up in stretching flat sheets are given straightforwardly as

$$\tau_{11} = \frac{4\mu \dfrac{\partial v_1}{\partial x_1}}{1 - \left(2\theta \dfrac{\partial v_1}{\partial x_1}\right)^2} \tag{3}$$

For the fluid used to obtain the picture given in Figure 1 the parameters are (*11*)

$$\theta \approx 0.1 \text{ sec.}$$

$$\mu \approx 3 \text{ poise}$$

at the low shear rate levels involved. Thus, Equation (3) predicts the stress to rise to infinity at a stretch rate

$$\frac{\partial v_1}{\partial x_1} \approx 5 \text{ sec.}^{-1}$$

Under the flow conditions used to obtain Figure 1 the stretch rates of the fluid at the entry to the tube may be obtained from the tabulated data (*11*); they are of the same magnitude.

Under these conditions, combination of Equations (1) and (3) gives, upon proper neglect of the shearing stress term $\tau_{r\theta}$ of Equation (1)

$$\frac{\partial F}{\partial r} = \frac{\partial}{\partial r}(-\tau_{rr}) = \frac{-4\mu \dfrac{\partial v_1}{\partial x_1}}{r\left[1 - \left(2\theta \dfrac{\partial v_1}{\partial x_1}\right)^2\right]} \tag{4}$$

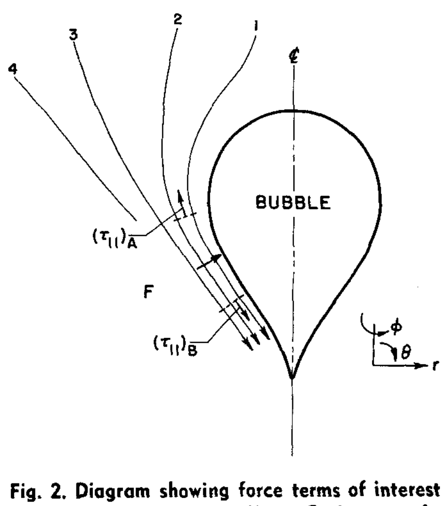

Fig. 2. Diagram showing force terms of interest in analysis of Uebler effect. *F* denotes the force per unit area applied by the tensile stresses within the fluid.

and will be very great in view of the fact that the term $2\theta(\partial v_1/\partial x_1)$ is approaching unity. Since the right-hand side is inherently negative, and since the force of interest F goes to zero as $r \rightarrow \infty$, (that is, there is no fluid motion far from the bubble in the infinite reservoir), as one integrates inward from an infinite radial position, Δr is also inherently negative, yielding a positive, finite value for the force F.

With reference to Figure 2, similar considerations apply to the upstream face of the bubble; therefore the net force will be in the upstream direction and will serve to hold the bubble or particle in place only if the stretch rate of the fluid changes sufficiently over the length of the bubble to counteract the drag forces of the fluid. This implies either relatively large bubbles or rapidly accelerating fluid stretch rates or some combination of these as prerequisites for the occurrence of the Uebler effect.

Some consideration of the effects of bubble size are in order. Astarita and Apuzzo (2) have shown that as the bubble size becomes larger, the drag forces on a bubble increase only slowly with increases in bubble size providing the bubble retains its streamlined shape. In view of the form of Equation (4) slight changes in the fluid stretching rate would appear to suffice for very large changes in the retarding force. Thus, there does not seem to be a limit on the size of the bubble which may be retarded by sufficiently rapidly stretching viscoelastic fluids, at least at this level of approximation. As the bubble size decreases, at a given flow rate of the viscoelastic fluid, it should be pushed upstream into a region of lower fluid acceleration rate and take up a new equilibrium position.

As the bubble size decreases to very small values its surface will no longer be free, with the gas inside acting inviscidly relative to the field fluid, but the interface instead becomes rigid (*1*, *2*). This results in a 50% increase in the drag coefficient over a small range of bubble diameters (*2*), implying that the stabilization of solid particles or of very small bubbles may require appreciably higher fluid stretching rates than necessary for larger ones. Additionally, as the bubble or particle size approaches zero, the rate-of-change of fluid stretching rate (with position in the velocity field) will necessarily become constant, and more rapidly accelerating velocity fields will be required to stop a bubble than if the bubble or particle diameter is large enough to enable it to take advantage of the nonlinearities inherent in the spatial derivative of the fluid stretch rate. These may be the reasons for the apparent inability of the polymeric flows studied to date to retard the motion of very small bubbles, but it is of interest to note that such effects have been noted elsewhere.

There is an alternate manner in which infinite stresses may also be predicted to occur at finite stretch rates. In the case of purely viscous fluids ($\theta = 0$) having an apparent viscosity function which approaches infinity at finite stresses, the same high normal stress magnitudes are predicted. This is precisely the behavior (dilatant or shear thickening) found to occur in certain highly concentrated suspensions (*9*, *10*), and Riggs (*10*), in a careful study, the results of which were shown to be independent of tube entry configuration, has noted that in such suspensions the solid particles are rejected at the tube entry at high flow rates, although they enter uniformly at low flows. This odd similarity in the behavior of highly elastic polymeric systems and inelastic but highly concentrated suspensions has been noted recently in other respects also (*8*).

CONCLUDING AND SUMMARIZING REMARKS

In rapidly converging velocity fields large tensile stresses may be set up in either viscoelastic or strongly dilatant fluids. The magnitude of the principal (axial) stress term is given by Equation (3) for fluids described by the convected Maxwell constitutive relationship, Equation (2); it may be shown that while the detailed result given in Equation (3) is dependent on the choice of the constitutive equation used, the general form of the equation for the stress is quite independent of the constitutive equation employed, and infinite stresses are predicted at finite stretch rates in all cases (*1*). It is of interest to note in passing that while the second-order approximation to simple fluid behavior does not predict infinite stresses at finite stretch rates, it is the incorrect asymptotic approximation under conditions of high stretch rate. Use of the full Rivlin–Ericksen approximation does, on the other hand, correctly enable one to obtain an expression of the form of Equation (3).

The shapes of the streamlines of the macroscopic flow field (Figure 2) appear to be dependent upon these maximum permissible fluid stretch rates. The result of these stresses is to generate forces which, when acting upon a foreign object in the converging flow field such as the bubble depicted in Figure 2, will tend to impede its motion and may stop it. Thus a clear separating effect is operative.

In the case of solid particles or very small bubbles whose behavior in a field fluid is equivalent to solid particles, the same forces are operative, but the shearing stress term $\tau_{r\theta}$ of Equation (1) may not be negligible. The result will be that such particulate materials will require more rapidly accelerating flow fields for their motion to be stopped, although under such more extreme flow conditions the same Uebler effect should be obtainable and has indeed been observed in highly concentrated suspensions.

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